MST

Put into CPME DETAIL

E4 BKG = 0.17 c/sec

DE45 = 0.1 c/sec.

E5 BKG = 0.07 c/sec.

DE56 = 0.04 c/sec.

E6 BKG = 0.03 c/sec.

H J

MST — 30 tapes put in

[PAM] — Definitely put in { Elect Bkg
{ Diff Elect. Calc

E3 GEOMETRY FACTOR (J only 0.22 $cm^2 sr$)

ELECTRON EFFICIENCIES

Basic data available from earlier calculations & input are energy/nuc. fluxes and thresholds in energy/nuc., energy/charge & rigidity

---

Problem: given $\psi(E/nuc)$ get $\psi'(E/chg)$

$\psi' = \psi$ protons

$\psi' = 2\psi$ alphas, mediums, heavies

Problem: given $\psi\,(E/nuc)$, get ψ' (rigidity)

$$R = \frac{pc}{ze} \qquad p = (2m_p A E)^{1/2} \qquad E = p^2/2m$$

$$p = (2ME)^{1/2}$$

$$R = \frac{(2m_p A E)^{1/2} c}{ze} = \frac{c\sqrt{2m_p}\,A}{ze}\left(\frac{E}{A}\right)^{1/2}$$

$$R = \frac{\sqrt{2m_p c^2}}{e}\left(\frac{A}{z}\right)\left(\frac{E}{A}\right)^{1/2}$$

$$\psi' = \underbrace{\frac{\sqrt{2m_o c^2}}{e}}_{1.14154\times10^{8}} \psi^{1/2} \quad \text{protons}$$

$$\psi' = \underbrace{2\,\frac{\sqrt{2m_o c^2}}{e}}_{2.28308\times10^{8}} \psi^{1/2} \quad \text{alphas, mediums heavies}$$

Problem: given $\frac{dj}{d\psi}(E/nuc)$ get $\frac{dj}{d\psi'}(E/chg)$

$$\frac{dj}{d\psi'} = \frac{dj}{d\psi} \quad \text{protons}$$

$$\frac{dj}{d\psi'} = 2\,\frac{dj}{d\psi} \quad \text{alphas, mediums, heavies}$$

gauss-cm
or ergs/esu

ψ = Energy/nuc ergs/nuc
ψ' = gauss cm

$$\frac{dj}{d\psi'} = \frac{Z}{A}\sqrt{2}\, e \underbrace{\left(\frac{\psi}{2m_p c^2}\right)^{1/2}}_{\text{dimensionless}} \cdot \frac{dj}{d\psi}$$

$$\frac{d}{d\psi} \rightarrow \frac{1}{\text{ergs/nuc}} \rightarrow \frac{1}{\text{meV/nuc}} \cdot \frac{\text{meV}}{\text{erg}}$$

1 MeV = 1.602×10^{-6} ERG

$$\underset{\text{ergs/nucl.}}{\frac{d}{d\psi}} \rightarrow 6.242 \times 10^{5} \underset{\text{meV/nucl}}{\frac{d}{d\psi}}$$

CPME Processing ✓ 9/18/72.

Calculation of Composition Ratios.

Objective: Compute j_p/j_α for each of the 6 alpha channels using the assumptions of proton spectra as power laws in a.) energy/nucleon
b.) energy/charge
c.) rigidity

From the assumed form for the proton spectrum, calculate the proton flux in the specified interval corresponding to the alphas using the 2 closest proton channels to determine the spectral parameters.

Let $\psi \equiv$ energy/nuc, energy/chg, rigidity as appropriate

$\psi_1^\alpha(i)$, $\psi_2^\alpha(i) \equiv$ lower & upper limits for i^{th} channel of α's

$\psi_1^P(i')$, $\psi_2^P(i') \equiv$ lower & upper limits for i'th channel of protons

$\psi_1^P(i'+1)$, $\psi_2^P(i'+1) \equiv$ lower & upper limits for $i'+1$ th channel of protons

$\frac{dj_\alpha}{d\psi}(i) \equiv$ flux of i^{th} alpha channel.

$\frac{dj_P}{d\psi}(i') \equiv$ flux of i'^{th} proton channel.

$\frac{dj_P}{d\psi}(i'+1) \equiv$ flux of $i'+1$ th proton channel

Problem: Given $\frac{dj}{d\psi}$ (E/nuc) get $\frac{dj}{d\psi'}$ (rigidity)

$$\frac{dj}{dR} = \frac{dj}{d(E/A)} \cdot \frac{d(E/A)}{dR}$$

$$\frac{dR}{d(E/A)} = \frac{\sqrt{2m_pc^2}}{e}\,\frac{A}{Z}\cdot\frac{1}{2}\left(\frac{E}{A}\right)^{-1/2}$$

$$\frac{dj}{d\psi'} = \underbrace{\frac{eZ2}{A\sqrt{2m_pc^2}}}\cdot\psi^{1/2}\frac{dj}{d\psi}$$ where $\frac{Z}{A} = 1$ for H^1

$\frac{Z}{A} = \frac{1}{2}$ for α's, M's & H's.

Steps to Compute ratio p/α.

1.) Find # alphas in i^{th} interval φ_i^α:

$$\varphi_i^\alpha \doteq \underbrace{\frac{dj^\alpha(E_i)}{dE}\cdot\left(E_2^\alpha(i) - E_1^\alpha(i)\right)}_{\text{all in } E/\text{nuc.}}$$

$N_i^\alpha \equiv$ # / cm² sec sr. in $E_1 \le E \le E_2$
<u>also</u> is # / cm² sec sr. in $\psi_1^\alpha(i) \le \psi \le \psi_2^\alpha(i)$
in E/charge & rigidity.

2.) Find # protons in interval $\psi_1^\alpha(i) \leq \psi \leq \psi_2^\alpha(i)$
given proton fluxes

$$\frac{dj^p(i')}{dE} \quad \& \quad \frac{dj^p(i'+1)}{dE}$$

where i', $i'+1$ are the two proton channels closest to the i th alpha channel.

a.) convert proton fluxes from E/nuc into E/chg. or rigidity as appropriate (see note above)

b.) obtain proton power law spectral exponent γ from

$$\gamma = \ln\left\{\frac{\frac{dj^p}{d\psi}(i'+1)}{\frac{dj}{d\psi}(i')}\right\} \Big/ \ln\left\{\frac{\psi^p(i'+1)}{\psi^p(i')}\right\}$$

where $\psi^p(i'+1)$ is midpoint of $i'+1$ interval
& $\psi^p(i')$ is midpoint of i' th interval

c.) obtain proton power law coefficient from

$$N_{i'}^p = \frac{dj^p(i')}{d\psi} \Big/ (\psi^p(i'))^\gamma$$

d.) Find # of protons between $4_1^{\alpha}(i)$ & $4_2^{\alpha}(i)$ by integrating the power law spectrum over the appropriate interval.

$$\varphi_i^{P} = \int_{4_1^{\alpha}(i)}^{4_2^{\alpha}(i)} d4 \; \frac{dj_P}{d4} = \frac{dj^{P}(i')}{d4} \cdot \frac{1}{(4^{P}(i'))^{\gamma}} \cdot \int_{4_1^{\alpha}(i)}^{4_2^{\alpha}(i)} d4 \; 4^{\gamma}$$

$$\gamma \neq -1 \quad \varphi_i^{P} = \frac{dj^{P}(i')}{d4} \cdot \left\{ \frac{\left(4_2^{\alpha}(i)\right)^{\gamma+1} - \left(4_1^{\alpha}(i)\right)^{\gamma+1}}{(\gamma+1)\,(4^{P}(i'))^{\gamma}} \right\}$$

$$\gamma = 1 \quad \varphi_i^{P} = \frac{dj^{P}(i')}{d4} \cdot \left\{ 4^{P}(i') \ln \frac{4_2^{\alpha}(i)}{4_1^{\alpha}(i)} \right\}$$

e.) Form abundance ratio

$$r_{p\alpha}(i) = \varphi_i^{P} / \varphi_i^{\alpha}$$

and uncertainty $\delta r_{p\alpha}(i) = \left\{ (\delta \varphi_i^{P})^2 + (\delta \varphi_i^{\alpha})^2 \right\}^{1/2}$.

where $\delta \varphi_i^{P}$ = unc. in flux · spectral factor

∴ multiplicative correction factors

suppose $\frac{\beta}{\alpha}$ & γ are known.

$$\frac{R_p}{R_A} = \frac{G_p N_p}{\cancel{(\gamma_p+1)}} \frac{\left(E_3^{p\,\gamma+1} - E_1^{p\,\gamma+1}\right)\cancel{(\gamma_\alpha+1)}}{G_A N_A \left(E_3^{\alpha\,\gamma+1} - E_1^{\alpha\,\gamma+1}\right)}$$

$$\frac{R_p}{R_t} = \frac{R_p}{R_p + R_A} = \frac{1}{1 + \frac{R_A}{R_p}} = \frac{R_p}{R_A}\left[\frac{1}{1 + \frac{R_p}{R_A}}\right]$$

$$\frac{R_p}{R_A} = \left(\frac{\beta}{\alpha}\right) \cdot \frac{\left(E_{\beta\,upper}^{\gamma+1} - E_{\beta\,lower}^{\gamma+1}\right)}{\left(E_{\alpha\,upper}^{\gamma+1} - E_{\alpha\,lower}^{\gamma+1}\right)}$$

Given $R = G \int_{E_1}^{E_2} dE \, N E^{\gamma}$ and γ

Find N

and $\frac{dj}{dE}(E_i) = N \, E_i^{\gamma}$

$\gamma \neq -1$

$$R = GN \frac{\{E_2^{\gamma+1} - E_1^{\gamma+1}\}}{(\gamma+1)}$$

$$\frac{dj(E_i)}{dE} = \frac{(\gamma+1) R}{G\{E_2^{\gamma+1} - E_1^{\gamma+1}\}} E_i^{\gamma}$$

$\gamma = -1$ $R = GN \ln E_2/E_1$

$$\frac{dj(E_i)}{dE} = \frac{R \quad E_i^{-1}}{G \ln E_2/E_1}$$

8/12/72

To: Krimigis, Wende, Kohl.

Re: "Special Event Search"

Action: Review & return any comments or additions to me by 25 Aug.

From: T P. Armstrong

5.0 ~~4.)~~ <u>Search for Special Events</u>

After the true count rates R_i and uncertainties δR_i are available, tests will be made looking for the signature of various physically interesting occurances.

5.1 ~~i.)~~ <u>Anisotropy studies</u> ~~P1, A1, E4, P11, P10, E3,~~ E1, E2A, E3, E4, P1, P8, A1, A7, and Z1 ~~E2A~~ are all sectored into 8 angular sectors in the snapshot in which they appear. All of the sectors are of equal size, 45^o except E3's which are 11.25^o, (sweeping through a 45^o angle in 8 snapshots) a difference which is not important here. Associated with each sector (k) for each detector (i) there is a true counting rate $R_i^{(k)}$ and an uncertainty $\delta R_i^{(k)}$. For each sectored detector each time it appears we apply the following test.

Is $\left| R_i^{(k+1)} - R_i^{(k)} \right| > 3 \left| \delta R_i^{(k)} + \delta R_i^{(k+1)} \right|$

and $R_i^{(k+1)} + R_i^{(k)} \neq 0$

and $\dfrac{\left| R_i^{(k+1)} - R_i^{(k)} \right|}{R_i^{(k+1)} + R_i^{(k)}} > 1/2$?

If yes, punch out a card with the information

sorting
flag

1, i, k, Album, pg, SS, U.T., R_i^k, R_i^{k+1}

and set a counter to loop around <u>all</u> anisotropy checks immediately for the next 10 albums, to avoid punching too many cards.

Let k run from 1 to 8 and define k = 9 as k = 1.

Omit solar sectors for the GM tubes E1, E2A, E3 (I do not know which these are but it's easy to find out later).

~~11.)~~ 5.2 Particle or X-ray Onsets

After 2 logical records have been input and are available, we form a ~~running~~ ~~4~~ 2 album average (this is the same interval as the summing called for in 12.) and could very well be a part of it) of the detectors M, P9, A2, P2, E2B, E4 (spin averaged) and of the solar sectors of E1 and E3. When a valid (having no more than 1 missing pt) average is available $R_{i\ av.} = \frac{1}{N} \sum_{j=1}^{N} R_i^{(j)}$

where $N \equiv$ # times the R_i reading appeared in 4 albums

$$\delta R_{i\ av.} = \frac{1}{\sqrt{N}} \left\{ \sum_{j=1}^{N} (\delta R_i{}^j)^2 \right\} ;$$

we then compare the current point with the average of the 4 preceding albums as:

If $\quad |R_i - R_{i\ av.}| > 3 |\delta R_i + \delta R_{i\ av}|$

and $\quad R_{i\ av.} \neq 0$

and $\quad \dfrac{R_i - R_{i\ av.}}{R_{i\ av.}} > 1$

we output a card in the format 2, i, blank, album, pg, SS, U.T., R_i, $R_{i\ av.}$

If a card is output, we set a counter to loop around the punch routine for the next 10 albums (we must keep on computing the running average, however).

~~iii.)~~ 5.3 Rapid Spectral Variations

Using a ~~running 4 album~~ 5.5 MINUTE average of detector rates R_i as in ii.) above for E1, E2A, P1, P2, we look for abrupt changes in the count rate ratios. E1, E2A, P1 are sectored so we first sum over sectors (omitting solar) and then average over 4 albums. For the sectored detectors,

$$R_i = \frac{1}{8}\sum_{j=1}^{8} R_i^{(k)} \quad \text{(sum over sectors)}$$

k

$$\delta R_i = \frac{1}{8}\left(\sum_{j=1}^{8} (\delta R_i^{(j)})^2\right)^{1/2}$$

We then compute ~~a 4 album average~~ 5.5 minute average

$$\bar{R}_i = \frac{1}{N}\sum_{k=1}^{N} R_i^{(k)}$$

and $\delta\bar{R}_i = \frac{1}{\sqrt{N}}\left(\sum_{k=1}^{N} (\delta R_i^{(k)})^2\right)^{1/2}$

We then form the ratio

$$\bar{Q}_1 = \frac{E2A}{E1} = \frac{\bar{R}_i}{\bar{R}_{i'}}$$ where i labels E2A and i' labels E1

(If $\bar{R}_{i'} = 0$, simply loop around this test).

~~unc. $\frac{E2A}{E1}$ =~~ $\delta\bar{Q}_1 = \left((\delta\bar{R}_i)^2 + (\delta\bar{R}_{i'})^2\right)^{1/2}$

Now form the ratio Q_1 for the current snapshot, calculating

~~Q_1,~~ Q_1 as above but using current point instead of the averaged one.

and ~~T~~test

If $|Q_1 - \bar{Q}_1| > 3\,(\delta Q_1 + \delta\bar{Q}_1)$

and $\bar{Q}_1 \neq 0$

and $\dfrac{|Q_1 - \bar{Q}_1|}{\bar{Q}_1} > .3$

Punch a card with the format 3, i, i', Album, pg, SS, U.T., Q_1, $\overline{Q}_1$ and set a counter to loop around this test for the next 10 albums.

Repeat the above procedure for P1/P2 $\equiv Q_2$ and punch out a card in the format 4, i, i', Album, pg, SS, U.T., Q_2, $\overline{Q}_2$.

Dr Armstrong

TO: S. M. Krimigis, C. D. Wende, J. W. Kohl

RE: "Special Event Search"

ACTION: Review & return any comments or additivies to me by 25 August, 1972

FROM: T. P. Armstrong

5.0 Search for Special Events

After the true count rates R_i and uncertainties δR_i are available, tests will be made looking for the signature of various physically interesting occurances.

5.1 Anisotropy studies E1, E2A, E3, E4, P1, P8, A1, A7, Z1 are all sectored into 8 angular sectors in the snapshot in which they appear. All of the sectors are of equal size, 45° except E3's which are 11.25°, (sweeping through a 45° angle in 8 snapshots) a difference which is not important here. Associated with each sector (k) for each detector (i) there is a true counting rate $R_i^{(k)}$ and an uncertainty $\delta R_i^{(k)}$. For each sectored detector each time it appears we apply the following test.

IS $R_i^{(k+1)} - R_i^{(k)} > 3\ \delta R_i^{(k)} + \delta R_i^{(k+1)}$

and $R_i(k+1) + R_i^{(k)} \neq 0$

and $\dfrac{R_i^{(k+1)} - R_i^{(k)}}{R_i^{(k+1)} + R_i^{(k)}} > 1/2$?

If yes, punch out a card with the information

sorting
flag

1, i, k, Album, pg, SS, U.T., R_i^{k}, R_i^{k+1}

and set a counter to loop around all anisotropy checks immediately for the next 10 albums, to avoid punching too many cards.

Let k run from 1 to 8 and define k=9 as k=1. Omit solar sectors for the GM tubes E1, E2A, E3 (I do not know which these are but it's easy to find out later).

5.2 Particle or X-ray Onsets

After 2 logical records have been input and are available, we form a 2 album average (this is the same interval as the summing called for in 12.) and could very well be a part of it) of the detectors M, P9, A2, P2, E2B, E4 (spin averaged) and of the solar sectors of E1 and E3. When a valid (having no more than 1 missing pt) average is available $R_{i\ av.} = \frac{1}{N} \sum_{j=1}^{N} R_i^{(j)}$ where N = # times the R_i reading appeared in 4 albums

$$\delta R_{i\ av.} = \frac{1}{\sqrt{N}} \left\{ \sum_{j=1}^{N} (\delta R_i^{\,j})^2 \right\};$$

we then compare the current point with the average of the 4 preceding albums as:

$$\text{If} \quad R_i - R_{i\ av.} > 3|\delta R_i + \delta R_{i\ av}|$$

$$\text{and } R_{i\ av.} \neq 0$$

$$\text{and} \left|\frac{R_i - R_{i\ av.}}{R_{i\ av.}}\right| > 1$$

we output a card in the format 2, i, blank, album, pg, SS, U.T., R_i, $R_{i\ av.}$

If a card is output, we set a counter to loop around the punch routine for the next 10 albums (we must keep on computing the running average, however).

5.3 Rapid Spectral Variations

Using a 5.5 minute average of detector rates R_i as in ii.) above for E1, E2A, P1, P2, we look for abrupt changes in the count rate ratios. E1, E2A, P1 are sectored so we first sum over sectors, (omitting solar) and then average over 4 albums. For the sectored detectors,

$$R_i = 1/8 \sum_{k-1}^{8} R_i^{(k)} \qquad \text{(sum over sectors)}$$

$$\delta R_i = \frac{1}{8} \left(\sum_{j=1}^{8} (\delta R_i^{(j)})^2 \right) 1/2$$

We than compute 5.5 minute average

$$\overline{R}_i = \frac{1}{N} \sum_{k=1}^{N} R_i^{(k)}$$

and $$\delta\overline{R}_i = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} (\delta R_i^{(k)})^2 \quad 1/2$$

We then form the ratio

$$\overline{Q}_1 = \frac{E2A}{E1} = \frac{\overline{R}_i}{\overline{R}_{i'}}$$ where i labels E2A and i' lables E1

(If $\overline{R}_{i'} = 0$, simply loop around this test).

Now form the ratio Q_1 for the current snapshot calculating

Q_1 as above but using <u>current</u> point instead of the <u>averaged</u> one.

and test

If $/Q_1 - \overline{Q}_1/ > 3\ (\delta Q_1 + \delta\overline{Q}_1)$

and $\overline{Q}_1 \neq 0$

and $$\frac{/Q_1 - \overline{Q}_1/}{\overline{Q}_1} > .3$$

Punch a card with the format 3, i, i', Album, pg, SS, U.T., Q_1, $\overline{Q}_1$ and set a counter to loop around this test for the next 10 albums.

5/27/71

Table I DATA LABELS & POSITIONS

APL-NAME	S/C Accum #	POSITION IN TM. READOUT R.O. SS, SEQ, FR, CHANNEL	DESCRIPTIVE NAME	ACCUM TIME
APL-R1	LR12a_2-6	ALL, 1, 2, 4B/6 & 7	M	1 SS.
APL-R2	LR12a_2-10	ALL, 1, 10, 4B/6 & 7	S	
APL-R3	LR12a_2-14	ALL, 2, 2, 4B/6 & 7	P9	
APL-R4	LR12a_2-18	ALL, 2, 10, 4B/6 & 7	P7	
APL-R5	LR12a_2-20	ALL, 2, 10, 4B/9 & 10	Z1	
APL-R6	LR12a_2-22	ALL, 3, 2, 4B/6 & 7	A7	
APL-R7	LR12a_2-26	ALL, 3, 10, 4B/6 & 7	A6	
APL-R8	LR12a_3-6	EVEN, 1, 4, 4B/6 & 7	A5	2SS
APL-R9	LR12a_3-10	ODD, 0, 8, 4B/6 & 7	A4	
APL-R10	LR12a_3-14	EVEN, 0, 8, 4B/6 & 7	x A3	
APL-R11	LR12a_3-18	ODD, 1, 4, 4B/6 & 7	A2	
APL-R12	LR10a_3-1	EVEN, 0, 4, 11 & 2 BITS 12	x P11	
APL-R13	LR10a_3-2	EVEN, 0, 4, 6B/12 & 4B/13	x P10	
APL-R14	LR10a_3-5	EVEN, 1, 4, 11 & 2B/12	x E4	
APL-R15	LR10a_3-6	EVEN, 1, 4, 6B/12 & 4B/13	x E5	
APL-R16	LR10a_3-9	ODD, 0, 8, 11 & 2B/12	E6	
APL-R17	LR10a_3-10	ODD, 0, 8, 6B/12 & 4B/13	E2B	
APL-R18	LR10a_3-13	EVEN, 0, 8, 11 & 2B/12	x E2C	
APL-R19	LR10a_3-14	EVEN, 0, 8, 6B/12 & 4B/13	x P2	
APL-R20	LR10a_3-17	ODD, 1, 4, 11 & 2B/12	P3	
APL-R21	LR10a_3-18	ODD, 1, 4, 6B/12 & 4B/13	P4	
APL-R22	LR10a_3-21	EVEN, 0, 12, 11 & 2B/12	x P5	
APL-R23	LR10a_3-22	EVEN, 0, 12, 6B/12 & 4B/13	x P6	
APL-R24	LR10a_3-25	ODD, 0, 12, 11 & 2B/12	P8	
APL-R25	LR10a_3-26	ODD, 0, 12, 6B/12 & 4B/13	Z2	
APL-Se1	APL Se-1 1_8-5_8	ALL, 2, 2, 0-4 & 11-15		
APL-Se2	APL Se-2 1_8-5_8	ALL, 2, 10, 0-4 & 11-15		
APL-Se3	APL Se-3 1_8-5_8	ALL, 3, 2, 0-4 & 11-15		
APL-Se4	APL Se-4 1_8-5_8	ALL, 3, 10, 0-4 & 11-15		
APL-DP	APL DP 3-21	EVEN, 0, 12, 1ST BIT OF Ch. 4		
APL-AP	AP #1	SSO, 1, 0, 4		

E1	E1	E1	E1
E3*	E2A	E3*	E2A
P1	E4	P1	E4
A1	P11	P10	A6
App**			

* E3 IS DIVIDED INTO 32 SUBSECTORS AND REPEATS WITH A 4PAGE PERIOD

** APP HAS 8 SUBCOMMUTATED SIGNALS: STARTING FROM PG 0 OF EVEN ALBUMS THEY

4 subsectors read out

E31A	—	pg. 0	SS1
B	—	pg. 0	SS3
C	—	pg 1	SS1
D	—	pg 1	SS3

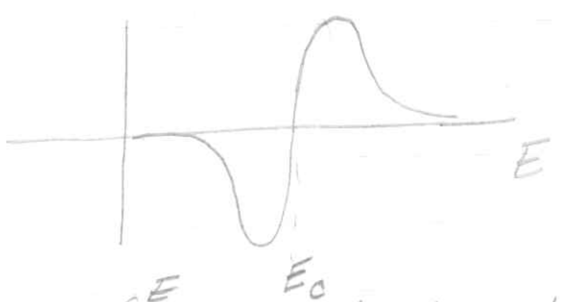
How to handle spectral fits

$$R_i = \int_0^\infty dE\ \epsilon_i(E)\, j(E) \qquad i = 1 \dots n$$

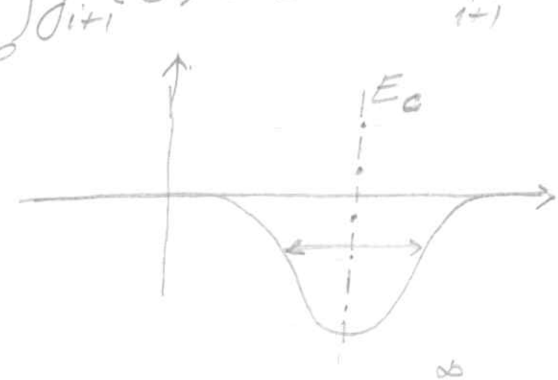
assume a form for $j(E)$

$$R_{i+1} - R_i = \int_0^\infty dE\ (\epsilon_{i+1}(E) - \epsilon_i(E))\, j(E)$$

suppose the function
$g_{i+1}(E) = \epsilon_{i+1}(E) - \epsilon_i(E)$ looks like



then $\int_{-\infty}^{E} g_{i+1}(\epsilon)\, dE' = h_{i+1}(\epsilon)$ looks like



$$R_{i+1} - R_i \equiv \Delta R_{i+1} = \int_0^\infty dE\ g_{i+1}(E)\ j(E)$$

$$= - \int_0^\infty dE\ h_{i+1}(E)\ \frac{dj(E)}{dE}$$

Begin spectral fits

$$j(E) = j(E_c) + (E-E_c)\left.\frac{dj(E)}{dE}\right|_{E=E_c} + \frac{(E-E_c)^2}{2!}\left.\frac{d^2 j(E)}{dE^2}\right|_{E=E_c} +$$

$$R_1 = \int_0^\infty dE\, j(E)\,\epsilon(E)$$

$$\int_0^\infty dE\,\epsilon(E) = K_1$$

$$R_i \doteq j(E_i) + \left.\frac{dj(E)}{dE}\right|_{E=E_i} \underbrace{\int_0^\infty dE\,\epsilon(E)\,(E-E_i)}_{K_2(E_i)} + \left.\frac{d^2 j(E)}{dE^2}\right|_{E=E_i} \underbrace{\int_0^\infty dE\,\epsilon(E)\frac{(E-E_i)^2}{2!}}_{K_3(E_i)}$$

$$R_i = j(E_i)K_1 + j'(E_i)K_2(E_i) + j''(E_i)K_3(E_i)$$

$$R_i \pm \delta R_i = j(E_i)K_1 + j'(E_i)K_2(E_i) + j''(E_i)K_3(E_i) \qquad i = 1 \text{ to } 11$$

step ① neglect 2^{nd} & 3^{rd} terms.

$$\frac{1}{K_1}(R_i \pm \delta R_i) = j_0(E_i) \qquad j(E_i) \qquad i = 1, \dots, N$$

② to the set of $j_0(E_i) \pm \delta j_0(E_i)$ $\quad i = 1, \dots N$.

③ one fits a smooth curve

a.) polynomial

b) exponential

or uses finite differences to estimate

$j'(E_i) \qquad i = 1, \dots N-1$

eg $\quad j'(E_1) = (j_0(E_2) - j_0(E_1)) \pm \left\{[\delta j_0(E_1)]^2 + [\delta j_0(E_2)]^2\right\}^{1/2}$

$$\therefore \quad j'(E_1) = j_0'(E_1) \pm \delta j_0'(E_1)$$

(4) Now correct the estimates for the $j(E_i)$

$$j_1(E_i) = j_0(E_i) \pm \delta j_0(E_i) - \frac{K_2(E_i)}{K_1(E_i)}\left(j_0'(E_i) \pm \delta j_0'(E_i)\right)$$

or $$j_1(E_i) = j_0(E_i) - \frac{K_2(E_i)}{K_1(E_i)} j_0'(E_i) \pm \underbrace{\left\{\left[\delta j_0(E_i)\right]^2 + \left[\frac{K_2}{K_1}\delta j_0'(E_i)\right]^2\right\}^{1/2}}_{\delta j_1(E_i)}$$

(5) Next approx. use $j_1(E_i)$ to get $j_1'(E_i)$ & $j_1''(E_i)$ with finite differences

$$j_1'(E_i) = j_1(E_{i+1}) - j_1(E_i) \pm \left\{\left[\delta j_1(E_{i+1})\right]^2 + \left[\delta j_1(E_i)\right]^2\right\}^{1/2}$$

$$j_1''(E_i) = j_1'(E_{i+1}) - j_1'(E_i) \pm \left\{\left[\delta j_1'(E_{i+1})\right]^2 + \left[\delta j_1'(E_i)\right]^2\right\}^{1/2}$$

$$j_2(E_i) = \frac{1}{K_1}(R_i \pm \delta R_i) - \frac{K_2(E_i)}{K_1(E_i)}\left(j_1'(E_i) \pm \delta j_1'(E_i)\right) - \frac{K_3(E_i)}{K_1(E_i)}\left(j_1''(E_i) \pm \delta j_1''(E_i)\right)$$

or $$j_2(E_i) = \frac{R_i}{K_1} - \frac{K_2}{K_1}(E_i)\, j_1'(E_i) - \frac{K_3(E_i)}{K_1} j_1''(E_i) \pm \underbrace{\left[\left\{\frac{\delta R_i}{K_1}\right\}^2 + \left\{\frac{K_2(E_i)}{K_1}\delta j_1'(E_i)\right\}^2 + \left\{\frac{K_3}{K_1}(E_i)\,\delta j_1''(E_i)\right\}^2\right]^{1/2}}_{\delta j_2(E_i)}$$

Consider some typical cases.

$$E \to 0, \quad \epsilon(E) \to 0$$

if $\epsilon(E) \xrightarrow[E \to \infty]{} 0$

& $\epsilon(E_c - E') = \epsilon(E_c + E')$

then $K_2 = 0$

I. "Ideal" passband

also $K_2 = \int_{-E_c}^{\infty} dE'\, E'\, \epsilon(E' + E_c)$

$$K_1 = \int_0^\infty dE\, \epsilon(E) = E_2 - E_1$$

$$K_2 = \int_0^\infty dE\, (E - E_c)\epsilon(E) = \frac{E}{2}^2 - E_c E \Big|_{E_1}^{E_2}$$

$$K_2 = \frac{E_2^2 - E_1^2}{2} - E_c(E_2 - E_1) = (E_2 - E_1)\left[\frac{E_1 + E_2}{2} - E_c\right]$$

Minimize K_2 wrt E_c $\qquad K_2 = 0$ if $E_c = \frac{E_1 + E_2}{2}$

$$\frac{\partial K_2}{\partial E_c} = -(E_2 - E_1) \Rightarrow \text{it has no extremum}$$

Look at K_3

$$K_3 = \int_0^\infty \frac{dE}{2!}(E - E_c)^2 \epsilon(E) = \frac{1}{2!}\left\{\int_{E_1}^{E_2} dE \left\{E^2 - 2EE_c + E_c^2\right\}\right\}$$

$$= \frac{1}{2!}\left\{\frac{1}{3}E^3 - E^2 E_c + E E_c^2\right\}\Big|_{E_1}^{E_2}$$

$$= \frac{1}{2!}\left\{\frac{E_2^3 - E_1^3}{3} - (E_2^2 - E_1^2)E_c + (E_2 - E_1)E_c^2\right\}$$

$$K_3 = \frac{E_2 - E_1}{2!}\left\{\frac{E_2^2 + E_2 E_1 + E_1^2}{3} - (E_1 + E_2)E_c + E_c^2\right\}$$

at the minimum of K_2 $\qquad E_c = \frac{E_1 + E_2}{2}$

$$K_3 = \frac{E_2 - E_1}{2!}\left\{\frac{E_2^2 + E_2 E_1 + E_1^2}{3} - \frac{(E_1 + E_2)^2}{2} + \frac{(E_1 + E_2)^2}{4}\right\}$$

$$K_3 = \frac{E_2 - E_1}{2!}\left\{E_2^2\left\{\frac{1}{3} - \frac{1}{2} + \frac{1}{4}\right\} + E_1^2\left\{\frac{1}{3} - \frac{1}{2} + \frac{1}{4}\right\} + E_1 E_2\left\{\frac{1}{3} - 1 + \frac{1}{2}\right\}\right\}$$

$$K_3 = \frac{E_2 - E_1}{2!}\left\{\frac{1}{12}(E_2^2 + E_1^2) - \frac{E_1 E_2}{6}\right\}$$

$$K_3 = \frac{E_2 - E_1}{2!} \cdot \frac{1}{12}\left\{E_2^2 + E_1^2 - 2E_1E_2\right\} = \frac{(E_2 - E_1)^3}{24}$$

<u>ok</u>
checked
by another
method

if $E_c = \frac{E_1 + E_2}{2}$ we have

$$K_1 = E_2 - E_1 \qquad \frac{K_2}{K_1} = 0 \qquad \frac{K_3}{K_1} = \frac{(E_2 - E_1)^3}{(E_2 - E_1)24} = \frac{(E_2 - E_1)^2}{24}$$

now suppose $j(E)$ is of the form

$$j(E) = \left(\frac{N_0}{E_0}\right)\left(\frac{E}{E_0}\right)^{-\gamma}$$

Not a good form

$$j'(E) = -\gamma \frac{N_0}{E_0}\left(\frac{E}{E_0}\right)^{-(\gamma+1)}$$

$$j''(E) = +\gamma(\gamma+1)\frac{N_0}{E_0^2}\left(\frac{E}{E_0}\right)^{-(\gamma+2)}$$

$$j''(E_c) = \gamma(\gamma+1)\frac{N_0}{E_0^2}\left(\frac{E_2 + E_1}{2E_0}\right)^{-(\gamma+2)}$$

how large is

$$e = \frac{\frac{K_2}{K_1} \cdot j''(E_c)}{j} = \frac{\frac{(E_2 - E_1)^2}{24}\frac{\gamma(\gamma+1)N_0}{E_0^2}\left(\frac{E_2 + E_1}{2E_0}\right)^{-(\gamma+2)}}{N_0\left(\frac{E_2 + E_1}{2E_0}\right)^{-\gamma}}$$

$$e = \frac{(E_2 - E_1)^2\gamma(\gamma+1)}{24E_0^2}\left(\frac{E_2 + E_1}{2E_0}\right)^2 = \frac{\gamma(\gamma+1)}{96}\frac{(E_2^2 - E_1^2)^4}{E_0^4}$$

other choices for E_c
minimize K_3

$$K_3 = \frac{1}{2!}\left\{\frac{1}{3}(E_2^3 - E_1^3) - (E_2^2 - E_1^2)E_c + (E_2 - E_1)E_c^2\right\}$$

$$\frac{\partial K_3}{\partial E_c} = \frac{1}{2!}\left\{-(E_2^2 - E_1^2) + 2(E_2 - E_1)E_c\right\}$$

$$\frac{\partial^2 K_3}{\partial E_c^2} = (E_2 - E_1) > 0$$

extremum $(E_2 - E_1)E_c = \frac{1}{2}(E_2^2 - E_1^2) = \frac{1}{2}(E_2 - E_1)(E_2 + E_1)$

$$E_c = \left(\frac{E_2 + E_1}{2}\right)$$

$\Rightarrow$ Same choice of $E_c = \frac{E_2 + E_1}{2}$ minimizes K_3

Suggested prescription for choosing E_c
- choose it so that $K_2 = 0$

& we have $j_2(E_i) = \frac{R_i}{K_1} - \frac{K_3(E_i)}{K_1} j_1''(E_1) \pm \left[\left(\frac{\delta R_i}{K_1}\right)^2 + \left(\frac{K_3}{K_1}(E_1)\delta j_1''(E_1)\right)^2\right]^{1/2}$

$i = 1, \ldots, N$

I. Steps
1. Define the set of $K_1(E_i)$ & $K_3(E_i)$ (experimental data put into the numerical quadrature
2. calculate background corrected counting rates $R_i \pm \delta R_i \mid i = 1, \ldots, N$
3. Decide how many of the $i = 1, \ldots N$ rates are useable - some criterion on $\frac{\delta R_i}{R_i}$.

4. Calculate $j_2(E_i) \pm \delta j_2(E_i)$ for all significant i's.
5. Fill in upper limits for non-significant i's
6. Fit some smooth curve in the least squares sense to the set of significant $j_2(E_i)$'s
7. Output the set of $j_2(E_i)$'s (all) + the paramcters of the smooth curve
8. Fit curves to the set of j_2's in terms of other paramiters
   a.) Energy / charge
   b.) Rigidity

Overall organization

1. Do I for the alphas (verify that the $Z \geq 3$'s are unimportant, calc. fluxes of $Z > 3$'s in the int. of observation)
2. Take alpha spectrum from 1. & compute "backgrounds" to be subtracted from proton channels
3. Do I for the protons
4. Use proton & alpha spectra from 1 & 2 to calculate correction for E4, E5, E6 & get out "true" R's ± δR
5. Calculate an <u>integral</u> electron spectrum from E4, E5, E6 using <u>a</u> technique similar to I. (fit a two parameter energy (~~[illegible]~~ or Rigidity) spec

6.) Calculate foreground proton & alpha contribution to E1, E2A, E3 & subtract it out

7.) Estimate the electron contribution to solar sector (s) of E1, E2A, E3

a.) use adjacent sectors of E1, E2A, E3 or.

optional [b.) estimate from the PET data on E4, E5, E6.

& calculate solar l ray rates

8.) Put solar l ray rates into C.D.W's spectrum calc.

probably should do this only on 4 snap. basis

9.) For non-solar sectors of E1, E2A, E3 form a spin averaged integral electron spectrum to go along with that from E4, E5, E6

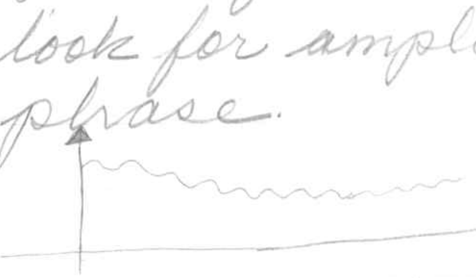
10.) use E1, E2A, E3, E4 (when available) to compute 2 parameter electron integral spectrum)

do this on a time average

11.) For galactic studies use a fourier analysis technique or signal averaging (superposed epoch)

(a.) assume chgd. flux does not have durable anisotropy features on 11° basis

(b.) fourier analyze a <u>long</u> series of E1 data and look for amplitude & phase.



<u>or</u> average the four subsectors
& take the difference & superpose
many sets of observations

or

as the source processes through the sector

Note: for a point source a time average is identical to an angle average

if $\epsilon(\alpha)$ is the detector effeciency vs angle to the collimator axis then

$$R_{sector}(\alpha) = \int_{\alpha - \frac{11.25}{2}}^{\alpha + \frac{11.25}{2}} d\varphi \, \epsilon(\varphi)$$

suppose $\epsilon(\varphi) = A(\varphi_0 - |\varphi|)$ $\quad$ & $\alpha > 0$

$$0 \leq \alpha < \frac{11.25}{2}$$

(per unit flux) $$R_{sector}(\alpha) = \int_{\alpha - \frac{11.25}{2}}^{0} d\varphi \, A(\varphi_0 + \varphi) + \int_{0}^{\alpha + \frac{11.25}{2}} d\varphi \, A(\varphi_0 - \varphi)$$

$$R_{sector}(\alpha) = A\left(\varphi_0 \varphi + \frac{\varphi^2}{2}\right)\Bigg|_{\alpha - \frac{11.25}{2}}^{0} + A\left(\varphi_0 \varphi + \frac{\varphi^2}{2}\right)\Bigg|_{0}^{\alpha + \frac{11.25}{2}}$$

$$R_{sector}(\alpha) = A\varphi\left(\varphi_0 + \frac{\varphi}{2}\right)\Big|_{\alpha - \frac{11.25}{2}}^{\alpha + \frac{11.25}{2}}$$

$$R_{sector}(\alpha) = A\varphi_0\left[11.25\right] + \frac{A}{2}\left[\left(\alpha + \frac{11.25}{2}\right)^2 - \left(\alpha - \frac{11.25}{2}\right)^2\right]$$

$$R_{sector}(\alpha) = A\varphi_0\left[11.25\right] + \frac{A}{\cancel{2}}\left[\cancel{2}\times 11.25\,\alpha\right]$$

$$R_{sector}(\alpha) = 11.25A(\varphi_0 - \alpha)$$

Since A & φ_0 are known then a measurement of R yields α if the X-ray flux has been determined. otherwise two measurements are needed.

12.) How to handle sectored data

assume $j(\varphi) = a_0 + a_1\cos(\varphi - \varphi_0) + b_1\sin(\varphi - \varphi_0)$
$+ a_2\cos 2(\varphi - \varphi_0) + b_2\sin 2(\varphi - \varphi_0)$
$+ a_3\cos 3(\varphi - \varphi_0) + b_3\sin 3(\varphi - \varphi_0)$

we must solve.

$$\left(a_0 + a_1\cos(\varphi_1 - \varphi_0) + b_1\sin(\varphi_1 - \varphi_0) + a_2\cos 2(\varphi_1 - \varphi_0) + b_2\sin 2(\varphi_1 - \varphi_0)\right)$$

OVER

=

a_0

Ben Ferrer
1.) assume nominal
2.)
3.)

do list

1. Find out what we can get on the GSFC experimenter's tape

assume that it is possible → a.) Will they uncompress the data?

Schedule Bill Barnes 1 week

b.)

2. See Roy about specifying where in the encoder output our data occurs

3. Specify output for input to Fortran program

$2^8 = 8 \cdot 2^5 = 256$

24 BIT LOG COMPRESSION.

	1	3	6	9	12	15	18	21
	X X X	X X X	X X X	X X X	X X X	X X X	X X X	X X X
	2^0	2^3	2^6	2^9	2^{12}	2^{15}	2^{18}	2^{21} 2^{22} 2^{23}
"0" SET	111	111	111	111	111	111	111	111
1	000	000	000	000	000	000	000	000

$\longrightarrow$ shift right & look for the 1st "1"

$N \equiv$ # shifts to get a "1" in the 2^{23} position

now the shifted contents are.

$$X_1\, X_2\, X_3\, X_4\, X_5\, X_6\, X_7\, 1$$

now X_7 <u>came</u> from the 2^{22-N} position of the accumulator, hence.

$$A_{cc} = 2^{23-N} + X_7 \cdot 2^{22-N} + X_6 \cdot 2^{21-N} + X_5 \cdot 2^{20-N} + X_4 \cdot 2^{19-N}$$

$$+ X_3 \cdot 2^{18-N} + X_2 \cdot 2^{17-N} + X_1 \cdot 2^{16-N} + \text{"}\Delta\text{"}$$

Δ → round-off term

$$\Delta = \frac{1}{2} \underbrace{(2^{16-N} - 1)}_{\text{max magnitude of truncated quantity}}$$

Max uncertainty is

$$\pm \frac{1}{2} (2^{16-N} - 1)$$

Suppose $N \leq 16$

then $\frac{unc}{A_{cc}} \leq \frac{1}{2} \frac{(2^{16-N} - 1)}{2^{23-N}}$ $\qquad N = 15$ $\quad \frac{1}{2} \cdot \frac{1}{2^8} = \frac{1}{2^9}$

% error varies from $\frac{1}{2^9}$ up to $\frac{1}{2^8}$

$\frac{1}{2^8} = 0.0039$. $\qquad \frac{1}{2^9} = 0.0018$.

$$\frac{1}{\sqrt{N}} = .05 \doteq \frac{1}{200}$$

$$R_i \pm \boxed{f\, R_i}$$

$$\overline{R} = \frac{1}{N}\sum_{i=1}^{N} R_i$$

$$\boxed{\sigma_{\overline{R}} \doteq \frac{f}{\sqrt{N}}\,\overline{R}}$$

$\sqrt{N} = 200$

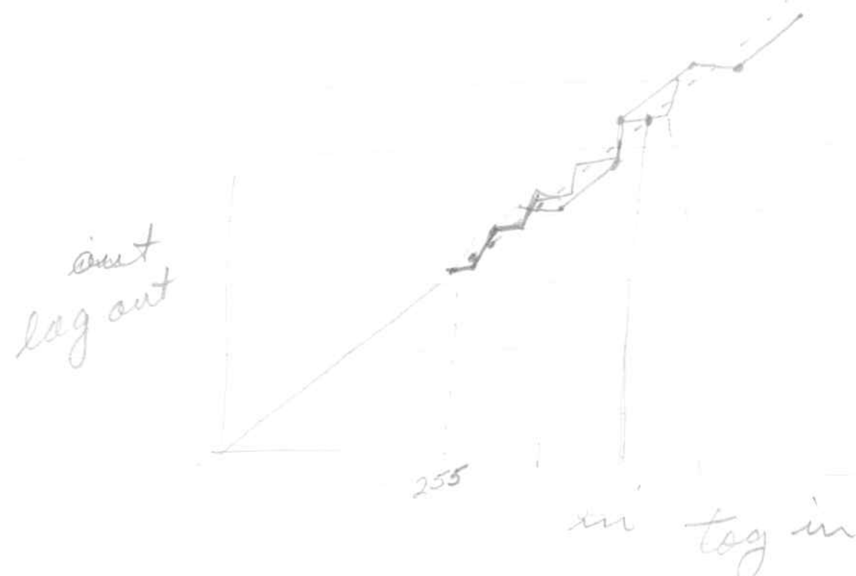
$N = 4 \times 10^4$

$$\frac{\sigma_{\overline{R}}}{\overline{R}} = \frac{f}{\sqrt{N}}$$

$.05$

$R_i \pm \sigma$

$$\sigma_{\overline{R}} = \frac{1}{\sqrt{N}}\,\sigma$$



1) This is the first set of operations to be done on the GSFC experiment tape. There is a maximum possible observed counting rate in each output that is set by rate limiting at a nominal 3×10^5 counts/sec in the case of the P.E.T. outputs and by the time constants of the GM tube circuits for E1, E2A, B, C & E3. Precise figures will be available after calibration of each unit. Typical GM tube maximum rates are in the range of 20 to 30 KHZ. The largest number which can be read out of 24/12 Bit log compressed accumulator is $2^{24} - 2^{15} = 16,744,448$ and of a 24/10 Bit $2^{24} - 2^{17} = 16,646,144$. Only the leading 4 digits are siginificant in the 24/12 bit case & the leading 3 in the 24/10 bit case. Accumulated count totals in excess of the maximum rate times the accumulation time can be identified and flagged as bad data, either by the quality flag or setting the rate equal to the "fill" number. No RUSR conversion will be

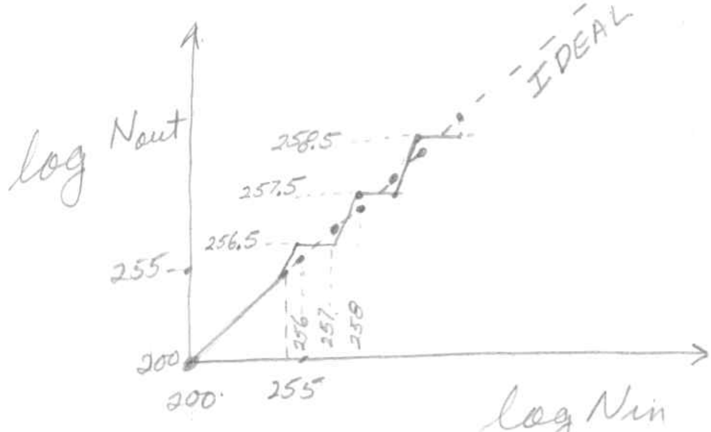
made on rates exceeding the max. rate.

Applying R vs r corrections will involve evaluating a function $R(r)$ where r is the apparent rate in counts/sec. from the data and R is a functional form <u>to be determined by laboratory calibration</u>.

The uncertainty in r for any channel will arise from two sources: one, the Poisson counting uncertainty of random events and two, the discretization uncertainty for the logarithmic compression. The Poisson counting uncertainty of R is $\pm\sqrt{N_{TOT}}$ where $N_{TOT.} = R\,T_{ACC}$ & $T_{ACC} \equiv$ accumulation time.

A.

Because of the system of discretization we have an input-output function like



24/12 bit compression

as long as the dispersion of values of N_{in} due to physical effects is larger than several steps – one can treat the discretization noise as "random". ~~or nearly as random as the variations in N_{in}~~. When the fluctuation in N_{in} is smaller than a step ($\frac{1}{2}$%) then one has a systematic error which can not be treated as a random one.

(Now if the physical fluctuation is less than a step (but not too much less) then for fixed avg. counts/sec one expects to see some occurance of values on adjacent steps and one could guess for a large number of observations the average value to somewhat better than $\frac{1}{2}$%

In the case that the variation of N_{in} is much less than ½% and greater than 255 counts then the precision of an average is no better than ½%, no matter how many observations are taken. In practice this is not an important effect and will not be further considered.)

~~MAX COUNTING RATE BEFORE compression sets in~~

Rate registers 24-12 bit read & set each SS (5.12 sec)

APL.	SIG.	ACC.	TIME.	MAX (C/SEC)
R1-R7	M, S, P8, P7, Z1, A7, A6	24-12 Bit	5.12	$\frac{255}{5.12} = 49.8$
R8-R11	A5, A4, A3, A2	24-12 BIT	10.24	$\frac{255}{10.28} = 24.9$
R12-R25	P11, P10, E4, E5, E6, E2B, E2C, P2, P3, P4, P5, P6, P8, Z2	24-10 bit	10.24	$\frac{63}{10.28} = 6.15$
S1-S4	E1, E2A, E3, E4, P1, P10, P11, A6	24-10	3 spins $\frac{3.75}{8}$	$\frac{63 \cdot 8}{3.75} = \underline{\underline{134}}\ \frac{c}{sec}$

M & S scintillators may have a rate from GCR's close to that necessary to get into the compression regime.

Algorithm for computing correct counting rates & uncertainties

Let N_i = counts from GSFC tape, not necessarily integer after logarithmic decompression

T_i = accumulation interval of the i^{th} output (= one or two snapshots or $\frac{1}{6}$, $\frac{3}{6}$, $\frac{3}{32}$ spins, see table I)

$r_i = N_i / T_i \equiv$ "apparent" counting rate

$R_i(r_i) \equiv$ "true" rate, functional form to be specified

Let δr_i be the discretization uncertainty in r_i. For

24/10 bit compression $\delta r_i = 0 \quad N_i \le 63$

$$\delta r_i = \tfrac{1}{2}\left(2^{\alpha_i - 5} - 1\right)/T_i$$

or For 24/12 bit compression

$$\delta r_i = 0 \quad N_i \le 255$$

$$\delta r_i = \tfrac{1}{2}\left(2^{\alpha_i - 7} - 1\right)/T_i$$

where α_i is the highest power of 2 present in r_i

We can estimate the uncertainty in R_i by assuming that δr_i & the statistical fluctuation in R_i add in an r.m.s. way to yield.

$$T_i\,\delta R_i = \left\{ R_i T_i + \left(\frac{\partial R_i}{\partial r_i}\right)^2 T_i^{\,2} (\delta r_i)^2 \right\}^{1/2}$$

$$\boxed{\delta R_i = \left\{ \frac{R_i}{T_i} + \left(\frac{\partial R_i}{\partial r_i}\right)^2 (\delta r_i)^2 \right\}^{1/2}}$$

where $\frac{\partial R_i}{\partial r_i}$ is evaluated at r_i

For each of the outputs R1–25 & Se1–4 the rate R_i & uncertainty δR_i must be computed according to the above procedure.

3. Considerations on time scale for analysis

A. Not all data is suitable for analysis on a single snapshot basis.

B. The obvious choice for Logical Record length is 4 snapshots = 1 page = 20.48 sec at 1600 bps or 81.92 sec. at 400 bps.

		Measurement	# times occuring in LR. (4SS)		for archives tape	Acc. time (total)
8	16	P1 (only sectored)	2	4	32	6 spins 7.5 sec.
1	2	P2	2	4	4	20.48
1	2	P3	2	4	4	"
1	2	P4	2	4	4	"
1	2	P5	2	4	4	"
1	2	P6	2	4	4	"
1	4	P7	4	8	8	"
1	2	P8	2	4	4	"
1	4	P9	4	8	8	"
1	2	P10	2	4	4	"
1	2	P11	2	4	4	"
8	8	A1 (only sectored)	1	2	16	3 spins 3.75 sec
1	2	A2	2	4	4	20.48
1	2	A3	2	4	4	20.48
1	2	A4	2	4	4	"
1	2	A5	2	4	4	"
	4	A6	4	8	8	"
1	4	A7	4	8	8	"
1	4	M	4	8	8	"
1	4	Z1	4	8	8	"

				# times app in 2pg	# words	
1	2	Z2	2	4	4	20.48
8	32	E1 (only sectored)	4	8	64	12 spins 15 sec
8	16	E2A (only sectored)	2	4	32	6 spins 7.5 sec
1	2	E2B	2	4	4	20.48
1	2	E2C	2	4	4	20.48
32	16	* E3 (only sectored)	2	4	32	6 spins 7.5 sec
1	2	E4	2	4	4	20.48
1	2	E5	2	4	4	"
1	2	E6	2	4	4	"
8	8	P10	1	2	16	3 spins 3.75
8	8	P11	1	2	16	3 spins 3.75
8	8	A6	1	2	16	3 spins 3.75
8	16	E4	2	4	32	6 spins 7.5

188 readings → if summed give 88 readings

4

152

* E3 has an extra level of subcommutation
* 4 so the effective repeat cycle
is twice the above table

on an 8 snap basis we have
2 × 172 + 32 = 376 readings

Question: How do we know which of
the 4 sub sectors E3 is in?
and how do we check it out
to assure the correct phase.

4. Suggestion for handling data use 1 album sums (81.92 sec – same as exp 35)

1 album = 4 pp. = 16 SS.

11 albums ⇒ 15 min

22 albums ⇒ 30 min

44 albums ⇒ 60 min = 1 hr sums

In 1 album we will have either 4, 8, or 16 separate measurements of each detector channel – in order to keep track of the scale of temporal fluctuations as well as to assure easy appraisal of ~~of~~ the data we need 3 numbers

ⓐ AVG RATE ⓑ unc. of average ⓒ $\frac{\text{max} - \text{min}}{\text{one acc time}}$ Range of fluct.

ⓓ #observations

this requires 88 · 4 = 352 "numbers" (words/album)
[600 words long

[if we use snapshot by snaphot 752. — 4×188
[about a factor of 2 compression

Since we are going to keep a "special events" file anyway — why not carve up the data into 4 album chunks (327.68 sec long 5 min)

⇒ gives a factor of ~ 8 compression of orig. data data.

4 albums = 16 pg = 64 SS

Analysis of Integral Spectral data (ELECTRONS)

Let $j(>E) = \int_E^{\infty} dE' j(E')$ where $j(E)$ is the differential energy spectrum

Typical passbands.

A.) ideal

$\epsilon(E) = 0 \quad E < E_1$
$\epsilon(E) = \epsilon \quad E \geq E_1$

B.) triangular

$\epsilon(E) = 0 \quad E < E_1$

$\epsilon(E) = \epsilon\left(1 - \frac{(E - E_1)}{E_2}\right)$

$E_1 < E < E_2$

$\epsilon(E) = 0 \quad E > E_2$

Because $j(E')$ is positive definite,

$$\frac{d\,j(>E)}{dE} = -j(E) < 0,$$ hence the integral spectrum is always a <u>falling</u> spectrum with increasing energy

Now $$R_i = \int_0^{\infty} dE\, j(E)\, \epsilon_i(E) = \int_{E_1}^{\infty} dE\, j(E)\, \epsilon_i(E)$$

~~Integrate once by parts~~

$$\cancel{R_i = \epsilon_i(E) \underbrace{\int dE\, j(E)}_{j(>E)} \Big|_{E_1}^{\infty} - \int_{E_1}^{\infty} dE \frac{d\epsilon_i(E)}{dE} \int dE\, j(E)}$$

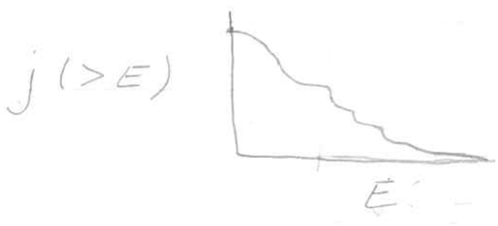
$$\cancel{R_i = \epsilon_i(E_1)\, j(>E_1) + \int_{E_1}^{\infty} dE\, j(>E_1) \frac{d\epsilon_i(E)}{dE}}$$

For integration by parts we can write

$$\int_a^b u \frac{dv}{dx}\, dx = uv\Big|_a^b - \int_a^b dx\, v \frac{du}{dx}$$

or $$\int_0^\infty dE\, \epsilon_1(E)\, j(E) = -\int_0^\infty dE\, \epsilon_1(E) \frac{dj(>E)}{dE}$$

$$R_i = \underbrace{-\epsilon_1(E)\, j(>E)\Big|_0^\infty}_{0} + \int_0^\infty dE\, j(>E) \frac{d\epsilon_i(E)}{dE}$$



$j(>E)$ must go monotically to zero. —

Suppose

$$j(>E) = N_1 e^{-E/E_0}$$

then $$\int_{E_i}^\infty dE\, N_1 e^{-E/E_0} \frac{d\epsilon_1(E)}{dE} = ?$$

take $\frac{d\epsilon_1(E)}{dE}$ from case B.

$$= \int_{E_1}^{E_2} dE N_1 e^{-E/E_0}\left(-\frac{\epsilon}{E_2}\right) = +\frac{N_1 \epsilon}{E_2} E_0 \int_{E_1}^{E_2} d\left(-\frac{E}{E_0}\right) e^{-E/E_0}$$

$$= \frac{N_1 \epsilon E_0}{E_2}\left(e^{-E_2/E_0} - e^{-E_1/E_0}\right) \Rightarrow \text{correction term}$$

0^{th} order term $= N_1 \epsilon e^{-E_1/E_0}$

$$\frac{\text{correction}}{0^{th}\text{ order}} = \frac{N_1 \epsilon E_0 \left(e^{-E_2/E_0} - e^{-E_1/E_0} \right)}{E_2 N_1 \epsilon e^{-E_1/E_0}}$$

$$= \frac{E_0}{E_2} \left\{ e^{\frac{-E_2+E_1}{E_0}} - 1 \right\}$$ Now the exponential factor will be negligible providing that $\frac{E_2 - E_1}{E_0} >> 1$

$$j_1(>E_1) = R_i \left\{ \frac{1}{\epsilon_i(E_i)} + \frac{E_0}{E_2} \left\{ e^{-(E_2-E_1)/E_0} - 1 \right\} \right\}$$

were E_0 is determined from the 0^{th} $\approx$ & ϵ E_1 & E_2 are from calibration data

Alternative method – numerically fit E1, E2A,B,C, E3 rates to an assumed spectrum & make a "best fit" analysis E4, E5, E6

?. Write $j(\varphi) = A_0 + \sum_{n=1}^{3} A_n \cos n(\varphi + \delta_n)$

$$j(\varphi) = A_0 + \sum_{n=1}^{3} A_n \{\cos n\varphi \cos n\delta_n - \sin n\varphi \sin n\delta_n\}$$

mult by 1 & integrate $0 < \varphi < 2\pi$

$$\int_0^{2\pi} d\varphi\, j(\varphi) = A_0$$

mult by $\cos\varphi$, $\sin\varphi$, etc

$$A_1 \cos\delta_1 = \frac{1}{\pi}\int_0^{2\pi} d\varphi\, j(\varphi)\cos\varphi$$

$$-A_1 \sin\delta_1 = \frac{1}{\pi}\int_0^{2\pi} d\varphi\, j(\varphi)\sin\varphi$$

or $\tan\delta_1 = \dfrac{-\int_0^{2\pi} d\varphi\, j(\varphi)\sin\varphi}{\int_0^{2\pi} d\varphi\, j(\varphi)\cos\varphi}$ $\qquad A_1 = \left[\frac{1}{\pi}\int_0^{2\pi} d\varphi\, j(\varphi)\cos\varphi\right]^2 + \left[\frac{1}{\pi}\int_0^{2\pi} d\varphi\, j(\varphi)\sin\varphi\right]^2$

$\tan\delta_2 = \dfrac{-\int_0^{2\pi} d\varphi\, j(\varphi)\sin 2\varphi}{\int_0^{2\pi} d\varphi\, j(\varphi)\cos 2\varphi}$ $\qquad A_2 = \left[\frac{1}{\pi}\int_0^{2\pi} d\varphi\, j(\varphi)\cos 2\varphi\right]^2 + \left[\frac{1}{\pi}\int_0^{2\pi} d\varphi\, j(\varphi)\sin 2\varphi\right]^2$

in general

$\tan\delta_n = \dfrac{-\int_0^{2\pi} d\varphi\, j(\varphi)\sin n\varphi}{\int_0^{2\pi} d\varphi\, j(\varphi)\cos n\varphi}$ $\qquad A_n = \left[\frac{1}{\pi}\int_0^{2\pi} d\varphi\, j(\varphi)\cos n\varphi\right]^2 + \left[\frac{1}{\pi}\int_0^{2\pi} d\varphi\, j(\varphi)\sin n\varphi\right]^2$

Now we must do integrals of the form

$$\int_0^{2\pi} d\varphi\, j(\varphi)\sin n\varphi \quad \text{or} \quad \int_0^{2\pi} d\varphi\, j(\varphi)\cos n\varphi$$

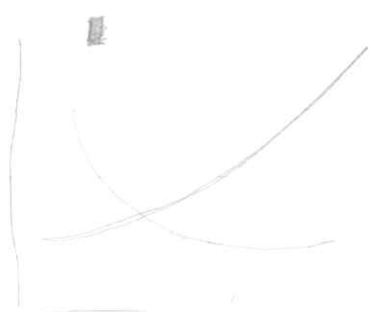
numerically, where we know $j(\varphi_i)$ $\quad i = 1 \cdots 8$

& $\varphi_{i+1} = \varphi_i + \pi/4$

- How about applying Swifts Mech to Solar Flare particle acc. ?

$\frac{\omega}{k} = v_{ph}$ $\omega \sim \omega_p^2 \left(1 + \frac{3}{2} k \langle v_{th} \rangle\right)$

$\frac{\omega}{k} \sim \frac{\omega_p}{k}\left(1 - \frac{3}{2} k \langle v_{th}^2 \rangle\right)$



$3^2/5$

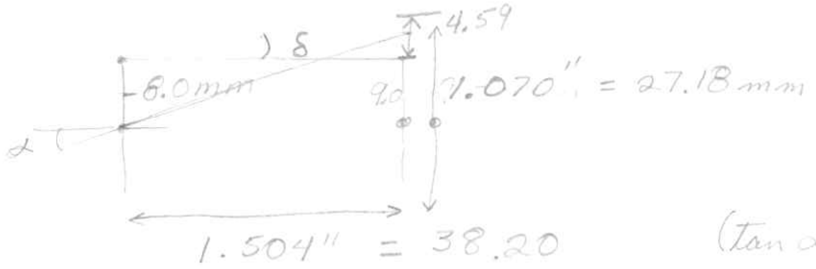
5.12
10.24
20.48
40.96
81.92 2^n
163.84
$\boxed{327.68}$

summing

$\boxed{25}$ $\boxed{\text{words}}$

D2 AREA $201\ mm^2 = \pi r_1^2$ $r_1 = \sqrt{63.980} = 8.00\ mm.$

D3 AREA $255\ mm^2 = \pi r_2^2$ $r_2 = \sqrt{81.17} = 9.00\ mm.$



$$\begin{array}{r} 2\,\underline{|27.18}\\ 13.59\\ \underline{9.00}\\ 4.59 \end{array}$$

$\left(\tan\alpha = \frac{11.3}{38.2} = 0.2958 = 16°30' \quad \cos\alpha = 0.9588\right)$

$$\tan\delta = \frac{4.59}{38.2} = 0.120 \qquad \delta = 7°$$

on edge. $0 < \delta < 7°$ satisfies

at $r = 4.0$ from center $\tan^{-1}\frac{5}{38.2} < \delta < \frac{9}{38.2}$

$$7° < \delta < 12°$$

quick & dirty

Ref. 5.5MeV(protons) σ goes as $\frac{1}{E^2}$

$$\int_{5°}^{10°} \sigma_{5.5MeV}(\theta)\, d\theta = 10.56 \times 10^{-28}\left(\frac{1}{\sin^2 5°} - \frac{1}{\sin^2 10°}\right)$$

$\sin 5° = 0.0872$ $\sin^2 5° = 0.0076$ $\frac{1}{7.6\times10^{-3}}$

$$\frac{1}{\sin^2 5°} = .13\times10^3 = 130$$

$$\frac{1}{\sin^2 10°} = \frac{1}{(.174)^2} = \frac{1}{.030} = 33$$

$$\begin{array}{r} 130\\ \underline{33}\\ 100 \end{array}$$

$$\sigma_{int} \sim 10^{-25}\ cm^2 = 0.1\ \underline{barn.}$$

1000.4 / 10

target nuclei $= 50\times10^{20} = 5\times10^{21}$

events/inc. part $= 10^{-25}\times 5\times10^{21} = \underline{\underline{5\times10^{-4}}}$

$^4 580.2 \ mm^2$

255.0

235.2

$\frac{235.2}{255.2} = .92$

6.023×10^{23} atoms/mole

aluminum $2.7 \ g/cm^3$

$\frac{6}{13} \cdot 2.7 \times 10^{23}$ atoms/cm^3

1.2×10^{23} atoms/cm^3

$$\langle \theta \rangle^2 \cong 4\pi \left(\frac{2zZe^2}{pv}\right)^2 \underbrace{\ln(210 Z^{-1/3})}_{13} \overbrace{tN}^{\# \text{targets}}$$

$$\left(\frac{zZe^2}{E}\right)^2$$

Aluminum $\quad t = 40$ mil $\quad t = \underset{2.54}{0.04}$ inches

$.1016$ cm

1.016 mm

$\sim 10^3$ microns.

micron $= 10^{-6}$ m

$= 10^{-4}$ cm

$= 10^{-3}$ mm

$pv = mv^2 = 2E$ nonrel.

$100 MeV = 10^8 eV = 1.6 \times 10^{-4}$ ERG

$e = 4.8 \times 10^{-10} \qquad e^2 = 23 \times 10^{-20}$

$z = 1 \quad Z = 13 \qquad zZe^2 = 299 \times 10^{-20}$

$\frac{zZe^2}{E} = \frac{299}{1.6} \times 10^{-16} = 187 \times 10^{-16}$

$\langle \theta^2 \rangle = 4\pi (1.87 \times 10^{-14})^2 \, 13 \, (tN)$

$\langle \theta^2 \rangle = 572 \times 10^{-28} \cdot tN \qquad N \cong 10^{23}/cm^3$

$= 5.72 \times 10^{-26} \cdot tN \qquad Nt \sim 10^{22}$

$\langle \theta^2 \rangle \cong 6 \times 10^{-4}$ RADIANS per 40 mil thickness ($\times 4$ to get to C)

$\langle \theta^2 \rangle_{tot} \cong 24 \times 10^{-4}$ radians

$\cong 1368 \times 10^{-4}$ degrees $= 0.137°$

Air path $\quad \frac{\Delta E}{\Delta x} \sim 10\,keV/cm$

$\underline{\quad 20 \quad}$

$200\,keV/cm$

$1.29 \times 10^{-3}\,g/cm^3$

6×10^{23} atoms $= 14\,g$

$1g = \frac{6}{14} \times 10^{23}$ atoms

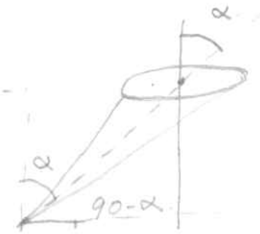
$\frac{6}{14} \cdot 1.29 \times 10^{20}$ atoms/cm³

$.55 \times 10^{20}$ atoms/cm³

$t \sim 20$

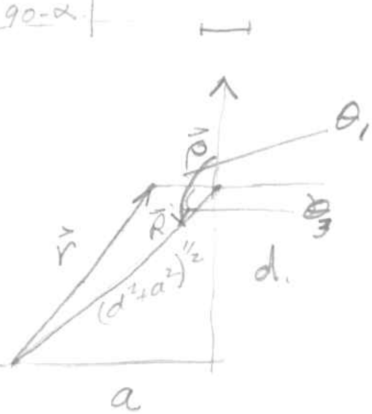
$Nt \sim 11.00 \times 10^{20}$

$\sim 1 \times 10^{21}$



$$\Omega = \int d\Omega = \int \frac{dA}{R^2} \cos\alpha$$

$$dA = \rho \, d\rho \, d\varphi$$



$$r^2 = \rho^2 + (a^2+d^2) - 2\,\vec{\rho}\cdot\vec{R}$$

$$R^2 = a^2 + d^2$$

$$\cos\theta = \cos\alpha_1 \cos\alpha_2 + \cos\beta_1 \cos\beta_2 + \cos\gamma_1 \cos\gamma_2$$

$\rho\,(\theta = \pi/2,\ \varphi)$

$$\cos\alpha_1 = \sin\theta\cos\varphi = \cos\varphi$$

$$\cos\beta_1 = \sin\theta\sin\varphi = \sin\varphi$$

$$\cos\gamma_1 = \cos\theta = 0$$

$$\cos\alpha_2 = \sin\theta\cos\varphi = \sin\theta_1$$

$$\cos\beta_2 = \sin\theta\sin\varphi = 0$$

$$\cos\gamma_2 = \cos\theta = \cos\theta_1$$

$$\sin\theta_1 = \frac{+a}{(d^2+a^2)^{1/2}} \qquad \cos\theta_1 = \frac{-d}{(d^2+a^2)^{1/2}}$$

$$\cos\theta_3 = \frac{a\cos\varphi}{(d^2+a^2)^{1/2}}$$

GEOMETRIC FACTOR OF INACTIVE RING (for particles passing through B)

$G \cong A \, \Delta\Omega \qquad A = 235.2 \text{ mm}$

$$\Delta\Omega \doteq (0.9588) \cdot \frac{201}{(11.3)^2 + (38.2)^2} = \frac{193}{1587} = 0.122 \text{ sr}$$

$$G \cong 2.35 \cdot 0.122 \text{ cm}^2 \text{ sr} = \boxed{0.287 \text{ cm}^2 \text{ sr}}$$

Geometric factor of $\underline{C}$ for particles passing through B. (201 mm²)

Vertex at center of C: $\Delta\Omega \cong \frac{201}{(38.2)^2} = 0.138$

Vertex at edge of C: $\Delta\Omega \cong \frac{201}{(38.2)^2 + (9.0)^2} \cong 0.138$

$$G \cong 2.35 \cdot 0.138 \text{ cm}^2 \text{ sr} = \boxed{0.324 \text{ cm}^2 \text{sr}}$$

I) long quiet run to get inelastic rate in Z1 & Z2 (set the lowest upper limit we can.

II

ΔE vs E complications

① electronic + channel noise
② var^{ns} in detector depth
③ landau Spread.

---

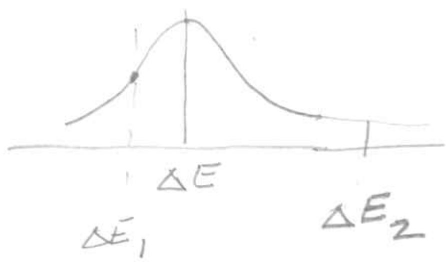
suppose $\Delta E\,(\text{ideal}) = f(E_{inc})$

$$\Delta E \pm \left\{ \delta^2_{LANDAU} + \delta^2_{THICKNESS} + \delta^2_{ELECTRONIC} + \delta^2_{ANGLE} \right\}^{1/2}$$

$E_{inc} = g$

select an E_{inc} compute $\Delta E \pm \delta$

probability that ΔE lies between ΔE_1 & ΔE_2



$$f(x) = e^{-\frac{(x-\Delta E)^2}{2\delta^2}}$$

$$\int_{x_1}^{x_2} dx\, f(x)$$

$\epsilon_i(E_{inc}) =$ Prob. of a count in channel i

Solid angle weighting isotropic flux

$$N(\alpha) = \int_0 d\Omega = \int d(\cos\theta)\, d\varphi$$

$$N(\alpha)$$

area on a spherical surface between α & $\alpha + d\alpha$ =

$$2\pi(a\sin\alpha)\, a\, d\alpha = 2\pi a^2 \sin\alpha\, d\alpha$$

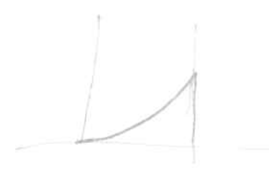
$$d\Omega = \frac{dA}{a^2} = 2\pi \sin\alpha\, d\alpha$$

$$\frac{d\Omega}{d\alpha} = 2\pi\sin\alpha$$

$$\int_0^{\alpha_c} 2\pi\sin\alpha\, d\alpha = -2\pi\cos\alpha \Big|_0^{\alpha_c} =$$

$$2\pi(1 - \cos\alpha_c)$$

$$\langle\alpha\rangle = 2\pi \frac{\int_0^{\alpha_c} \alpha\sin\alpha\, d\alpha}{2\pi(1-\cos\alpha_c)}$$



$$= \frac{\sin\alpha - \alpha\cos\alpha \Big|_0^{\alpha_c}}{(1-\cos\alpha_c)}$$

$$= \frac{\sin\alpha_c - \alpha_c\cos\alpha_c}{(1-\cos\alpha_c)} = \frac{0.191 - 0.189}{0.018} = \frac{0.002}{0.018}$$

$\alpha_c = 0.192$

suppose $\alpha_c = 11^\circ$ $\sin\alpha_c = 0.191$

$\cos\alpha_c = 0.982$

$\langle\alpha\rangle = 0.111$ radians $= 6.35^\circ$

$$\langle(\alpha-\bar{\alpha})^2\rangle = \frac{2\pi\int_0^{\alpha_c}(\alpha-\bar{\alpha})^2\sin\alpha\, d\alpha}{2\pi\,(1-\cos\alpha_c)}$$

$$= \frac{\int_0^{\alpha_c}\alpha^2\sin\alpha\, d\alpha - \int_0^{\alpha_c}2\bar{\alpha}\,\alpha\sin\alpha\, d\alpha + \int_0^{\alpha_c}\bar{\alpha}^2\sin\alpha\, d\alpha}{(1-\cos\alpha_c)}$$

$$\int_0^{\alpha_c}\alpha^2\sin\alpha\, d\alpha = 2\alpha\sin\alpha - (\alpha^2-2)\cos\alpha\Big|_0^{\alpha_c}$$
$$= 2\alpha_c\sin\alpha_c - (\alpha_c^2-2)\cos\alpha_c - 2$$

$$2\bar{\alpha}\int_0^{\alpha_c}\alpha\sin\alpha\, d\alpha = 2\langle\bar{\alpha}\rangle^2(1-\cos\alpha_c)$$

$$\int_0^{\alpha_c}\bar{\alpha}^2\sin\alpha\, d\alpha = \bar{\alpha}^2(-\cos\alpha)\Big|_0^{\alpha_c} = \bar{\alpha}^2(1-\cos\alpha_c)$$

$$\langle(\alpha-\bar{\alpha})^2\rangle = \frac{2(\alpha_c\sin\alpha_c-1)-(\alpha_c^2-2)\cos\alpha_c}{(1-\cos\alpha_c)} - 2\bar{\alpha}^2 + \bar{\alpha}^2$$

$$= \frac{2(\alpha_c\sin\alpha_c-1)-(\alpha_c^2-2)\cos\alpha_c}{(1-\cos\alpha_c)} - \bar{\alpha}^2$$